8) Actinium is a highly radioactive element. The most common isotope of actinium ($^{227}\mathrm{Ac}$) is produced as a by-product in nuclear reactors, and has a half-life of 21.77 years.

a) Obtain an exponential decay model for actinium-227 in the form $Q(t) = Q_0 e^{-kt}$. (Round k to four decimal places.)

Solution: You need to find k. Do this using the formula $k = \frac{\ln\left(\frac{1}{2}\right)}{\text{half-life}} = \frac{\ln\left(\frac{1}{2}\right)}{21.77} = -0.0318$. The model is $Q(t) = Q_0 e^{-0.0318t}$.

b) About 20 milligrams of actinium are produced in a certain nuclear reactor. Use your model to predict how long it will take for this amount of actinium to decay to one milligram.

Solution: Solve the equation $1 = 20e^{-0.0318t}$ for t. Divide both sides by 20 to get $\frac{1}{20} = e^{-0.0318t}$ and then take the natural log of both sides to get $\ln\left(\frac{1}{20}\right) = -0.0318t$. Lastly, divide both sides by -0.0318 to get $t = \frac{\ln(1/20)}{-0.0318} = 94.2$ years.

9) Find the equation of the exponential function passing through the points $(2, 9)$ and $(4, 20.25)$.

Solution: You need to find a function of the form $y = Ab^x$ that passes through the given points. Plug both points into the function to get $9 = Ab^2$ and $20.25 = Ab^4$. Divide one of these equation by the other to get

$$\frac{20.25}{9} = \frac{Ab^4}{Ab^2}$$

Simplify this and get $2.25 = b^2$, so $b = \sqrt{2.25} = 1.5$. The function is $y = A(1.5)^x$, now plug in one of the given points to find A: $9 = A(1.5)^2$, so $A = \frac{9}{1.5^2} = 4$, and the function is $y = 4(1.5)^x$.

10) There were 3,500 bacteria in a Petri dish (at time $t = 0$ hours). Four hours later, there were 5,500 bacteria in the dish. Find the mathematical model that represents the number of bacteria after t hours. It's an exponential formula of the form $Q(t) = Q_0 e^{kt}$.
Round k to 4 decimal places. Include the units in the answer.

Solution: We are given the initial number of bacteria, so we know $Q_0 = 3500$. We just have to find k. Do this by plugging in the other quantity at time $t = 4$: $5500 = 3500e^{k(4)}$, so $e^{4k} = \frac{11}{7}$, and $4k = \ln\left(\frac{11}{7}\right)$, then $k = \frac{1}{4}\ln\left(\frac{11}{7}\right) = 0.1130$. The function is $Q(t) = 3500e^{0.1130t}$.